Interference Phenomena and Spin Rotation of Neutrons by Magnetic Materials.

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Summary. — By the use of a neutron interferometer it is possible to study the transformation properties of spinors. The behaviour of neutrons in homogeneous and helical magnetic fields is studied. Intensity and polarization of the forward neutron beam after the interferometer, which is composed of an unchanged wave and a wave which has passed through a magnetic material, are calculated. Attention is paid to adiabatic helical fields. In all cases the influence of the neutron-nucleus potential is taken into account.

1. – Introduction.

With the recently developed neutron interferometer (1,2) the phase changes of particle waves can be measured. The present paper develops the theoretical basis for experiments, where the transformation properties of a spin represented by a spinor in magnetic fields are investigated. The neutron beam is split in the interferometer in two coherent parts, which are widely separated from each other. So it is possible to introduce in one beam path materials or fields—generally called phase shifters—which do not affect the second beam. These two partial beams afterwards interfere again and leave the interferometer in a

(1) H. RAUCH, W. TREIMER and U. BONSE: *Phys. Lett.*, **47** A, 369 (1974).
(2) W. BAUSPIESS, U. BONSE, H. RAUCH and W. TREIMER: *Zeits. Phys.*, **271**, 177 (1974).

forward beam, which is parallel to the incident beam, and a beam deviated by Laue scattering. For the ideal interferometer the two partial waves which come via the two beam paths and form the forward beam are identical when no phase shifter is used. Because of the close analogy to optics, we first develop the index-of-refraction formalism of a ferromagnetic material for neutrons.

2. – Homogeneous magnetic fields as phase shifters.

2'1. *The index of refraction of a ferromagnetic material for neutrons.* – For calculating at first the behaviour of neutrons in a homogeneous field, the index of refraction of a ferromagnetic material will be necessary. This index of refraction can be derived by means of the dynamical theory of magnetic neutron diffraction which was recently developed [3-5]. The potential energy of a low-energy neutron can be written as

$$U_{\text{pot}} = U_{\text{N}} - (\boldsymbol{\mu}\cdot\boldsymbol{B}), \tag{1}$$

where

$$U_{\text{N}}(\boldsymbol{r}) = (2\pi\hbar^2 b_c/M) \sum_j \delta(\boldsymbol{r} - \boldsymbol{r}_j) \tag{2}$$

is the nuclear part. Here, M is the mass of the neutron, b_c the coherent scattering length, $\boldsymbol{r}_j$ the position vector of the j-th nucleus, and $\boldsymbol{B}$ the magnetic induction. For the magnetic-dipole moment of the neutron

$$\boldsymbol{\mu} = \gamma\mu_{\text{N}}\boldsymbol{\sigma} = \mu\boldsymbol{\sigma} \tag{3}$$

holds, where μ_{N} is the nuclear magneton, μ the magnetic moment and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the Pauli vector of the neutron,

$$\begin{cases} \mu_{\text{N}} = e_0\hbar/2M_{\text{p}} = 5.0508\cdot10^{-27}\,\text{J/T}, \qquad \gamma = g_{\text{sn}}/2 = -1.91315, \\ \mu = \gamma\mu_{\text{N}} = -9.6630\cdot10^{-27}\,\text{J/T} = -6.0311\cdot10^{-8}\,\text{eV/T}. \end{cases} \tag{4}$$

The energy of the neutron in the vacuum is

$$E = (\hbar^2/2M)\,\boldsymbol{k}^2, \tag{5}$$

$\boldsymbol{k}$ being the wave vector in the vacuum. Thus, we obtain the Schrödinger

(3) H. Rauch and D. Petrascheck: Atominstitut-Report AIAU 74405 (1974).
(4) J. Sivardière: *Acta Cryst.*, A **31**, 340 (1975).
(5) H. H. Schmidt and P. Deimel: *J. Phys. C*, **8**, 1991 (1975).

equation within the medium:

(6) $$\{-(\hbar^2/2M)(\nabla^2+k^2)+U_N(\boldsymbol{r})-(\boldsymbol{\mu}\cdot\boldsymbol{B})\}\,\psi(\boldsymbol{r})=0\,.$$

If we solve this equation, the index of refraction, that is defined as

(7) $$n_\pm=k_{0\pm}/k$$

(with $k_{0\pm}=|\boldsymbol{k}_{0\pm}|$, $\boldsymbol{k}_{0\pm}$ being the wave vector in the medium, $\pm$ denoting the spinor components parallel and antiparallel to the magnetic field, $k=|\boldsymbol{k}|$), is obtained to be [6]

(8) $$n_\pm=[1-(U_0\mp\mu B_0)/E]^{\frac{1}{2}}\approx 1-(U_0\mp\mu B_0)/2E$$

with

(9) $$U_0=(2\pi\hbar^2/M)\,Nb_c \quad\text{and}\quad B_0=|\boldsymbol{B}_0|\,,\qquad \boldsymbol{B}_0=\boldsymbol{H}+4\pi\boldsymbol{M}\,.$$

Here N is the number of lattice points per volume element, $\boldsymbol{H}$ is the applied field and $\boldsymbol{M}$ is the magnetization of the sample.

2'2. *Phase factors of a spinor due to magnetic materials.* – In the following discussion we investigate the influence of a homogeneous magnetized magnetic material as a phase shifter on the waves emerging from the neutron interferometer. For waves in the forward direction the relation

(10) $$\psi_0^{\mathrm{I}}=\psi_0^{\mathrm{II}}$$

holds [7]. Here I and II denote the two beam paths within the interferometer. When a phase shifter is introduced into the first beam path we have, at first, to calculate its reflectivity

(11) $$(n-1)^2/(n+1)^2\approx[(2\pi\hbar^2/M)\,Nb_c\mp\mu B_0]^2/16E^2\,,$$

which is of the order of 10^{-10} to 10^{-9} for neutrons with $\lambda=0.5\ \text{nm}=5\ \text{Å}$, $k=12.57\ \text{nm}^{-1}$, $E=3.272\ \text{meV}$. Therefore we can neglect the waves travelling in the backward direction and, using the conditions of continuity at the two boundary surfaces, we obtain for the first partial wave behind the interferometer

(12) $$\psi_{0\pm}^{\mathrm{I}}=\psi_{0\pm}^{\mathrm{II}}\exp\left[i(kD)(n_\pm-1)\right].$$

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[6] I. I. GUREVICH and L. V. TARASOV: *Low Energy Neutron Physics* (Amsterdam, 1968).
[7] H. RAUCH and M. SUDA: *Phys. Stat. Sol.*, a **25**, 495 (1974).

Here D is the thickness of the phase shifter. The two equations (12) can be combined into

$$(13)\qquad \psi_0^{\mathrm{I}} = \exp\left[(ikD/2)[n_+ + n_- - 2 + (n_+ - n_-)\sigma_z]\right]\psi_0^{\mathrm{II}} = \exp\left[(iD/k)[-2\pi N b_c + (M\mu B_0/\hbar^2)\sigma_z]\right]\psi_0^{\mathrm{II}} = \exp[i\chi]\exp[is_z\varphi/\hbar]\,\psi_0^{\mathrm{II}}\,,$$

where $s_z = (\hbar/2)\sigma_z$ is the component of the neutron spin in the direction of the magnetic field. On the one hand, we obtain a phase shift

$$(14)\qquad \chi = -N\lambda b_c D$$

due to coherent neutron-nucleus scattering and, on the other hand, the magnetic field causes a rotation of the neutron spin by the angle

$$(15)\qquad \varphi = 2DM\mu B_0/\hbar^2 k = (D\mu B_0/\hbar)(2M/E)^{\frac{1}{2}} = -4.1897\cdot 10^5 (D/\mathrm{m})(B_0/\mathrm{T})(E/\mathrm{meV})^{-\frac{1}{2}}\,,$$

the axis of rotation being parallel to the direction of the magnetic field. With $D = 3$ cm, $B_0 = 10^{-3}\,\mathrm{T} = 10^{-3}\,\mathrm{Vsm}^{-2}$ ($\simeq 10$ Gauss), $E = 2.272$ meV the angle of rotation is $\varphi = -398°$. A rotation of the spin by an angle of $\varphi = 2\pi$

$$\exp[is_z\varphi/\hbar] = \exp[\pi i\sigma_z] = \cos\pi + i\sigma_z\sin\pi = -1$$

results in a destructive interference of the two partial waves. In order that the intensity, the phase and the direction of polarization attain their initial values again an angle of rotation of at least 4π is necessary

$$\exp[is_z\varphi/\hbar] = \cos 2\pi + i\sigma_z \sin 2\pi = 1\,.$$

The problem of observability of 2π rotations was discussed to some extent in the literature [8-10] and several experiments were proposed [8,11,12]. But only recently the effect was successfully measured [13] using the Laue-case neutron

(8) Y. AHARANOV and L. SUSSKIND: *Phys. Rev.*, **158**, 1237 (1967).
(9) G. C. HEGERFELDT and K. KRAUS: *Phys. Rev.*, **170**, 1185 (1968).
(10) G. T. MOORE: *Amer. Journ. Phys.*, **38**, 1177 (1970).
(11) H. J. BERNSTEIN: *Phys. Rev. Lett.*, **18**, 1102 (1967).
(12) A. G. KLEIN and G. I. OPAT: *Phys. Rev. D*, **11**, 523 (1975).
(13) H. RAUCH, A. ZEILINGER, G. BADUREK, A. WILFING, W. BAUSPIESS and U. BONSE: *Phys. Lett.*, **54 A**, 425 (1975).

interferometer. In that experiment the rotation was obtained by the field of an electromagnet in one beam path of the interferometer.

2'3. *The intensity behind the interferometer.* – If we introduce the initial intensity

$$I_i = 4\psi_0^{\mathrm{II}\dagger}\psi_0^{\mathrm{II}} \tag{16}$$

and the initial polarization

$$\boldsymbol{P}_i = \psi_0^{\mathrm{II}\dagger}\boldsymbol{\sigma}\psi_0^{\mathrm{II}}/\psi_0^{\mathrm{II}\dagger}\psi_0^{\mathrm{II}},$$

the wave function in the final state (after interference of the two partial beams) is

$$\psi_f = \{1 + \exp[i\chi]\exp[i\sigma_z\varphi/2]\}\psi_0^{\mathrm{II}}. \tag{17}$$

Therefore, the intensity in the final state is

$$\begin{aligned} I_f = \psi_f^\dagger\psi_f = 2\psi_0^{\mathrm{II}\dagger}\left(1 + \cos\chi\cos\frac{\varphi}{2} - \sigma_z\sin\chi\sin\frac{\varphi}{2}\right)\psi_0^{\mathrm{II}} = \\ = \frac{1}{2}I_i\left(1 + \cos\chi\cos\frac{\varphi}{2} - P_{iz}\sin\chi\sin\frac{\varphi}{2}\right), \end{aligned} \tag{18}$$

or, generally,

$$\begin{aligned} I_f = \frac{1}{2}I_i\left[1 + \cos\chi\cos\frac{\varphi}{2} - (\boldsymbol{P}_i\cdot\boldsymbol{B}/B)\sin\chi\sin\frac{\varphi}{2}\right] = \\ = \frac{1}{2}I_i\left\{\left[1 - \frac{\boldsymbol{P}_i\cdot\boldsymbol{B}}{B}\right]\cos^2\left(\frac{\chi}{2} - \frac{\varphi}{4}\right) + \left[1 + \frac{\boldsymbol{P}_i\cdot\boldsymbol{B}}{B}\right]\cos^2\left(\frac{\chi}{2} + \frac{\varphi}{4}\right)\right\}. \end{aligned} \tag{19}$$

For neutrons initially unpolarized or polarized perpendicularly to the direction of the magnetic field we obtain

$$I_f = \frac{1}{2}I_i\left(1 + \cos\chi\cos\frac{\varphi}{2}\right) \qquad \text{for } \boldsymbol{P}_i\cdot\boldsymbol{B} = 0. \tag{20}$$

For neutrons with polarization parallel to the field the intensity is

$$I_f = I_i\cos^2\left(\frac{\chi}{2} \pm \frac{\varphi}{4}\right) \qquad \text{for } \boldsymbol{P}_i\cdot\boldsymbol{B} = \pm B. \tag{21}$$

Now we consider the case where the sample with constant thickness is no longer homogeneously magnetized, but constituted of different domains with different directions of magnetization. We limit ourselves to domains with the same thickness as the whole sample and Bloch walls parallel to the neutron beam, so that the neutrons do not have to pass through a Bloch wall. When we divide the neutron beam in smaller beams, each of them traversing only one domain, we obtain for the j-th domain from eq. (19) the intensity

$$I_{fj} = \frac{1}{2} I_i \left\{1 + \cos\chi \cos\frac{\varphi}{2} - (\boldsymbol{P}_i \cdot \boldsymbol{B}_j/B_j) \sin\chi \sin\frac{\varphi}{2}\right\}. \tag{22}$$

By the use of the neutron interferometer it is therefore possible to map the magnetic domains in transmission through the sample directly. The imaging process can be made in a way similar to neutron radiography using an appropriate converter. If we choose the thickness of the sample and the strength of the magnetic field so that

$$\sin\chi = \sin\frac{\varphi}{2} = 1 ,$$

eq. (22) reduces to

$$I_{fj} = I_i \sin^2(\alpha_j/2) , \qquad \alpha_j = \sphericalangle (\boldsymbol{B}_j, \boldsymbol{P}_i) . \tag{23}$$

The intensity is zero for the polarization being parallel to the field, and attains its maximum value for the two vectors being antiparallel.

2'4. *The polarization behind the interferometer.* – The final polarization, that is the polarization after interference of the two neutron beams, is given by the vector

$$\boldsymbol{P}_f = \psi_f^\dagger \boldsymbol{\sigma} \psi_f / I_f . \tag{24}$$

Inserting the spinor ψ_f from eq. (18), we get

$$\boldsymbol{P}_f = \psi_0^{\mathrm{II}\dagger} \left[1 + \exp[-i\chi]\left(\cos\frac{\varphi}{2} - i\sigma_z \sin\frac{\varphi}{2}\right)\right] \boldsymbol{\sigma} \cdot \left[1 + \exp[i\chi]\left(\cos\frac{\varphi}{2} + i\sigma_z \sin\frac{\varphi}{2}\right)\right] \psi_0^{\mathrm{II}} / I_f .$$

The evaluation leads to the polarization vector

$$\boldsymbol{P}_f = \left[\left(P_{ix}\cos\frac{\varphi}{2} + P_{iy}\sin\frac{\varphi}{2}\right)\frac{\cos\chi + \cos(\varphi/2)}{1 + \cos\chi\cos(\varphi/2)} , \left(P_{iy}\cos\frac{\varphi}{2} - P_{ix}\sin\frac{\varphi}{2}\right)\frac{\cos\chi + \cos(\varphi/2)}{1 + \cos\chi\cos(\varphi/2)} , P_{iz} - \sin\chi_0\right] \Big/ (1 - P_{iz}\sin\chi_0) , \tag{25}$$

where the angle χ_0 is given by

$$\sin\chi_0 = \frac{\sin\chi\sin(\varphi/2)}{1 + \cos\chi\cos(\varphi/2)} = \frac{\cos^2(\chi/2 - \varphi/4) - \cos^2(\chi/2 + \varphi/4)}{\cos^2(\chi/2 - \varphi/4) + \cos^2(\chi/2 + \varphi/4)} . \tag{26}$$

With $\sin\chi = 0$ the vector $\boldsymbol{P}_f$ obtains from the vector $\boldsymbol{P}_i$ through a rotation around the z-axis—the direction of the magnetic field—by an angle of $\varphi/2$:

$$\boldsymbol{P}_f = \left(P_{ix}\cos\frac{\varphi}{2} + P_{iy}\sin\frac{\varphi}{2},\ P_{iy}\cos\frac{\varphi}{2} - P_{ix}\sin\frac{\varphi}{2},\ P_{iz}\right) \qquad \text{for } \sin\chi = 0\,, \tag{27}$$

the degree of polarization remaining unchanged:

$$P_f = P_i \qquad \text{for} \quad \sin\chi = 0\,. \tag{28}$$

When the initial polarization is parallel to the magnetic field, the polarization vector remains unchanged

$$\boldsymbol{P}_f = (0, 0, \pm 1) \qquad \text{for} \quad \boldsymbol{P}_i = (0, 0, \pm 1)\,. \tag{29}$$

If the initial polarization vanishes, the vector of the final polarization has the direction of the magnetic field:

$$\boldsymbol{P}_f = (0, 0, -\sin\chi_0) \qquad \text{for} \quad \boldsymbol{P}_i = 0\,. \tag{30}$$

In a recent experiment the effect of simultaneous nuclear phase shift and magnetic spin rotation was measured ([14]) with unpolarized incident neutrons. The « beat effect » of the intensity as predicted in eq. (20) was observed as well as the polarization of the forward-diffracted neutron beam.

3. – Helical magnetic fields as phase shifters.

In the real experimental situation the magnetic fields are very often varying in space. A special case of variation in space is a helical variation which will be considered here. As is well known, these helical fields arise in metamagnets on the one hand and on the other hand the variation of magnetization in domain walls can be described as helical to a good approximation. Special interest will be deserved to adiabatic helical fields, thus showing explicitly the analogy of spin rotations by homogeneous fields and adiabatic rotations in space.

3·1. *Solution of the Schrödinger equation.* – The direction of the neutron beam passing through the phase shifter with the magnetic field

$$\boldsymbol{B} = (\sin qy, 0, \cos qy)\, B_0 \tag{31}$$

([14]) G. BADUREK, H. RAUCH, A. ZEILINGER, W. BAUSPIESS and U. BONSE: *Phys. Lett.*, **56** A, 244 (1976); *Phys. Rev. D*, in press.

is assumed to be parallel to the y-axis. The Schrödinger equation within the phase shifter is

$$[(\hbar^2/2M)(\nabla^2+k^2)-U_N+(\boldsymbol{\mu}\cdot\boldsymbol{B})]\,\psi=0\,. \tag{32}$$

Here we use the spin matrices

$$\sigma_x=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\qquad \sigma_y=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},\qquad \sigma_z=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}.$$

With the ansatz

$$\psi=\begin{pmatrix}A_+\exp[i(p-q/2)y]\\ A_-\exp[i(p+q/2)y]\end{pmatrix} \tag{33}$$

we obtain for the amplitudes A_+ and A_- the homogeneous system of equations

$$\begin{cases}\{(\hbar^2/2M)[k^2-(p-q/2)^2]-U_N\}\,A_+ +\mu B_0 A_-=0\,,\\ \{(\hbar^2/2M)[k^2-(p+q/2)^2]-U_N\}\,A_- +\mu B_0 A_+=0\,.\end{cases} \tag{34}$$

Here we assume again that we are far away from any Bragg peak. The system of equations (34) only then has a nontrivial solution when its determinant vanishes. This condition leads us to two solutions for the wave number p:

$$p_{1,2}=\left\{k^2+\frac{q^2}{4}-u^2\pm[b^4+q^2(k^2-u^2)]^{\frac12}\right\}^{\frac12} \tag{35}$$

with the two wave numbers

$$\begin{cases}u=(2MU_N/\hbar^2)^{\frac12}=(4\pi Nb_c)^{\frac12}=(2k\chi/D)^{\frac12}\,,\\ b=(2M|\boldsymbol{\mu}|B_0/\hbar^2)^{\frac12}=(k\varphi/D)^{\frac12}\,.\end{cases} \tag{36}$$

For obtaining physically realizable approximations of the amplitudes A_+ and A_-, we have to estimate the order of magnitude of the wave numbers involved. For wavelengths $\lambda=(0.2\div 0.6)$ nm we have $k=(1\div 3)\cdot 10^{10}\,\text{m}^{-1}$, (the nuclear potential gives $u=(1\div 5)\cdot 10^7\,\text{m}^{-1}$. Magnetic fields of the value $|\boldsymbol{B}|=(10^{-2}\div 10^{-1})$ T give a wave number $b=(0.5\div 1.7)\cdot 10^7\,\text{m}^{-1}$. Finally, for $q=(1\div 4)\pi/D$ and $D=3$ cm we obtain $(qk)^{\frac12}=(0.2\div 0.4)\cdot 10^7\,\text{m}^{-1}$ and $q=(2\div 4)\cdot 10^2\,\text{m}^{-1}$. Therefore we get the relations

$$q\ll(qk)^{\frac12}<b\lesssim u\ll k\,. \tag{37}$$

Within this approximation eqs. (34) can be solved by means of eqs. (35). In this way we find for the spinor of eq. (33) the two orthogonal and stationary solutions

$$(38)\quad \begin{cases} \psi_1 = A_0 \begin{pmatrix} (\sqrt{b^4 + q^2k^2} + qk) \exp[i(p_1 - q/2)y] \\ - b^2 \exp[i(p_1 + q/2)y] \end{pmatrix}, \\ \psi_2 = A_0 \begin{pmatrix} b^2 \exp[i(p_2 - q/2)y] \\ (\sqrt{b^4 + q^2k^2} + qk) \exp[i(p_2 + q/2)y] \end{pmatrix} \end{cases}$$

with

$$(39)\quad \begin{cases} A_0 = [2\sqrt{b^4 + q^2k^2}(\sqrt{b^4 + q^2k^2} + qk)]^{-\frac{1}{2}}, \\ p_1 = k - \{u^2 - [b^4 + q^2(k^2 - u^2)]^{\frac{1}{2}}\}/2k, \\ p_2 = k - \{u^2 + [b^4 + q^2(k^2 - u^2)]^{\frac{1}{2}}\}/2k. \end{cases}$$

The phase shifter is assumed to lie in the interval $0 < y < D$ and to be oriented orthogonally to the direction of the incident neutrons. Again we can neglect the reflected waves. Therefore the spinor ahead, within and behind the phase shifter has the form

$$(40)\quad \begin{cases} \psi = (A_\alpha \alpha + A_\beta \beta) \exp[iky] & \text{for } y < 0, \\ \psi = A_1 \psi_1 + A_2 \psi_2 & \text{for } 0 < y < D, \\ \psi = (B_\alpha \alpha + B_\beta \beta) \exp[iky] & \text{for } D < y, \end{cases}$$

with the unit spinors obeying

$$\sigma_y \alpha = \alpha, \qquad \sigma_y \beta = -\beta, \qquad \alpha^\dagger \alpha = \beta^\dagger \beta = 1, \qquad \alpha^\dagger \beta = \beta^\dagger \alpha = 0.$$

The amplitudes A_α and A_β are given through the initial values of the intensity and the polarization vector. The amplitudes A_1, A_2, B_α, B_β are to be determined by means of the continuity conditions for the spinor at $y = 0$ and $y = D$. This procedure leads us to the relations

$$(41)\quad \begin{cases} A_1 = A_\alpha \sin\frac{\varphi_1}{2} - A_\beta \cos\frac{\varphi_1}{2}, \\ A_2 = A_\alpha \cos\frac{\varphi_1}{2} + A_\beta \sin\frac{\varphi_1}{2}, \end{cases}$$

and

$$(42)\quad \begin{cases} B_\alpha = \left(A_\alpha \cos\frac{\varphi_1}{2} + A_\beta \sin\frac{\varphi_1}{2}\right)\cos\frac{\varphi_1}{2}\exp[i(p_2 - k - q/2)D] - \\ \qquad - \left(-A_\alpha \sin\frac{\varphi_1}{2} + A_\beta \cos\frac{\varphi_1}{2}\right)\sin\frac{\varphi_1}{2}\exp[i(p_1 - k - q/2)D]\,, \\ B_\beta = \left(A_\alpha \cos\frac{\varphi_1}{2} + A_\beta \sin\frac{\varphi_1}{2}\right)\sin\frac{\varphi_1}{2}\exp[i(p_2 - k + q/2)D] + \\ \qquad + \left(-A_\alpha \sin\frac{\varphi_1}{2} + A_\beta \cos\frac{\varphi_1}{2}\right)\cos\frac{\varphi_1}{2}\exp[i(p_1 - k + q/2)D]\,, \end{cases}$$

where the angle φ_1 is defined by

$$\cos\frac{\varphi_1}{2} = A_0 b^2\,, \qquad \sin\frac{\varphi_1}{2} = A_0(\sqrt{b^4 + q^2k^2} + qk)\,.$$

We can combine eqs. (42) in the form

$$(43)\qquad \begin{pmatrix} B_\alpha \\ B_\beta \end{pmatrix} = U \begin{pmatrix} A_\alpha \\ A_\beta \end{pmatrix}, \qquad \text{or} \qquad \psi^{\mathrm{I}} = U\psi^{\mathrm{II}}\,,$$

where the transformation matrix U can be expressed by a product of exponential functions:

$$(44)\quad U = \exp[i\chi]\exp[-i\sigma_y\varphi_3/2]\exp[-i\sigma_x\varphi_1/2]\exp[i\sigma_y\varphi_2/2]\exp[i\sigma_x\varphi_1/2]\,.$$

Here, the phases are given by

$$(45)\qquad \begin{cases} \chi = (p_1 + p_2 - 2k)D/2 = -N\lambda b_c D\,, \\ \varphi_1 = 2\arccos A_0 b^2\,, \\ \varphi_2 = (p_2 - p_1)D = -\sqrt{\varphi^2 + (qD)^2}\,, \\ \varphi_3 = qD\,. \end{cases}$$

The matrix U can be written also in the form

$$(46)\qquad U = \exp[i\chi]\exp[-i\sigma_y\varphi_3/2]\exp[i\sigma_{y'}\varphi_2/2]\,,$$

the y'-axis resulting from the y-axis by a rotation around the x-axis by the angle φ_1. The phase shift χ is again due to the nuclear potential. The magnetic field produces a combined rotation of the direction of the spin around the y'-axis (angle of rotation φ_2) and the y-axis (angle of rotation φ_3). In the limit of a homogeneous field we have

$$q = 0\,,\qquad A_0 = 1/b^2\sqrt{2}\,,\qquad \varphi_1 = \pi/2\,,$$
$$\varphi_2 = -\,|\varphi| = \varphi\,,\qquad \varphi_3 = 0\,,\qquad y' = z\,,$$
$$U = \exp[i\chi]\exp[i\sigma_z\varphi/2]$$

in accordance with eq. (13).

3'2. *Intensity and polarization with a helical field.* – The initial intensity and the initial polarization are defined again by

(47) $$I_i = 4\psi_0^{\mathrm{II}\dagger}\psi_0^{\mathrm{II}}\,,\qquad \boldsymbol{P}_i = 4\psi_0^{\mathrm{II}\dagger}\boldsymbol{\sigma}\psi_0^{\mathrm{II}}/I_i\,.$$

That part of the forward beam which has passed via beam path I through the phase shifter is described by the spinor

(48) $$\psi_0^{\mathrm{I}} = U\psi_0^{\mathrm{II}} = \exp[i\chi]\,[\alpha_0 + i(\alpha_1\sigma_x + \alpha_2\sigma_y + \alpha_3\sigma_z)]\,\psi_0^{\mathrm{II}}\,.$$

The numbers

(49) $$\begin{cases} \alpha_0 = \cos\dfrac{\varphi_2}{2}\cos\dfrac{\varphi_3}{2} + \sin\dfrac{\varphi_2}{2}\sin\dfrac{\varphi_3}{2}\cos\varphi_1\,, \\[2ex] \alpha_1 = \sin\dfrac{\varphi_2}{2}\sin\dfrac{\varphi_3}{2}\sin\varphi_1\,, \\[2ex] \alpha_2 = -\cos\dfrac{\varphi_2}{2}\sin\dfrac{\varphi_3}{2} + \sin\dfrac{\varphi_2}{2}\cos\dfrac{\varphi_3}{2}\cos\varphi_1\,, \\[2ex] \alpha_3 = \sin\dfrac{\varphi_2}{2}\cos\dfrac{\varphi_3}{2}\sin\varphi_1 \end{cases}$$

obey the relation

(50) $$\sum_{i=0}^{3}\alpha_i^2 = 1\,.$$

Superposition of the two waves gives us the spinor of the forward beam in the final state:

(51) $$\psi_f = \psi_0^{\mathrm{I}} + \psi_0^{\mathrm{II}} = (1 + U)\,\psi_0^{\mathrm{II}}\,.$$

Thus, the intensity in the final state is

$$I_f = \psi_f^\dagger\psi_f = \psi_0^{\mathrm{II}\dagger}(1 + U^\dagger)(1 + U)\,\psi_0^{\mathrm{II}} =$$
$$= \tfrac{1}{2}I_i[1 + \alpha_0\cos\chi - (\alpha_1 P_{ix} + \alpha_2 P_{iy} + \alpha_3 P_{iz})\sin\chi]\,,$$

or

$$I_f = \tfrac{1}{2} I_i [1 + \alpha_0 \cos\chi - (\boldsymbol{\alpha}\cdot\boldsymbol{P}_i)\sin\chi] \tag{52}$$

with $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$.

For initially unpolarized neutrons the intensity is

$$I_f = \tfrac{1}{2} I_i (1 + \alpha_0 \cos\chi) \qquad \text{for } \boldsymbol{P}_i = 0. \tag{53}$$

For a total angle of rotation of the magnetic field in the phase shifter of 2π or 4π we obtain different intensities:

$$\begin{cases} I_f(\varphi_3 = 2\pi) = \\ \quad = \dfrac{1}{2} I_i \left[1 - \cos(\varphi_2/2)\cos\chi + (P_{iy}\cos\varphi_1 + P_{iz}\sin\varphi_1)\sin\dfrac{\varphi_2}{2}\sin\chi\right], \\ I_f(\varphi_3 = 4\pi) = \\ \quad = \dfrac{1}{2} I_i \left[1 + \cos(\varphi_2/2)\cos\chi - (P_{iy}\cos\varphi_1 + P_{iz}\sin\varphi_1)\sin\dfrac{\varphi_2}{2}\sin\chi\right]. \end{cases} \tag{54}$$

In both cases the intensity for initially unpolarized neutrons is the same as for an initial polarization in the x-direction. The polarization in the final state is given by the vector

$$\boldsymbol{P}_f = \psi_f^\dagger \boldsymbol{\sigma} \psi_f / I_f = \psi_0^{\mathrm{II}\dagger}(1 + U^\dagger)\boldsymbol{\sigma}(1 + U)\psi_0^{\mathrm{II}}/I_f. \tag{55}$$

Evaluation of the product (55) gives for the three components

$$\begin{cases} P_{fx} = (I_i/2I_f)[-\alpha_1 \sin\chi + (\alpha_1^2 + \alpha_0^2 + \alpha_0\cos\chi) P_{ix} + \\ \qquad + (\alpha_1\alpha_2 + \alpha_0\alpha_3 + \alpha_3\cos\chi) P_{iy} + (\alpha_1\alpha_3 - \alpha_0\alpha_2 - \alpha_2\cos\chi) P_{iz}], \\ P_{fy} = (I_i/2I_f)[-\alpha_2 \sin\chi + (\alpha_2\alpha_1 - \alpha_0\alpha_3 - \alpha_3\cos\chi) P_{ix} + \\ \qquad + (\alpha_2^2 + \alpha_0^2 + \alpha_0\cos\chi) P_{iy} + (\alpha_2\alpha_3 + \alpha_0\alpha_1 + \alpha_1\cos\chi) P_{iz}], \\ P_{fz} = (I_i/2I_f)[-\alpha_3 \sin\chi + (\alpha_3\alpha_1 + \alpha_0\alpha_2 + \alpha_2\cos\chi) P_{ix} + \\ \qquad + (\alpha_3\alpha_2 - \alpha_0\alpha_1 - \alpha_1\cos\chi) P_{iy} + (\alpha_3^2 + \alpha_0^2 + \alpha_0\cos\chi) P_{iz}], \end{cases} \tag{56}$$

where

$$2I_f/I_i = 1 + \alpha_0\cos\chi - (\boldsymbol{\alpha}\cdot\boldsymbol{P})\sin\chi$$

according to eq. (52). For an initially unpolarized neutron beam we get the polarization in the final state as

$$\boldsymbol{P}_f = \frac{-\sin\chi}{1+\alpha_0\cos\chi}\,\boldsymbol{\alpha} \qquad \text{for } \boldsymbol{P}_i = 0\,. \tag{57}$$

3·3. *The adiabatic case.* – In the adiabatic limit the spin of the neutron rotates together with the magnetic field in such a way that the angle between the magnetic field and the neutron spin remains constant. The magnetic field is so strong that we can write in the inequality relation (37)

$$qk \ll b^2\,. \tag{58}$$

With the above values of the numerical estimate the adiabatic limit is obtained for $B = 0.1\text{T}$. For the first of eqs. (39) we get $A_0 = 1/b^2\sqrt{2}$ and herewith it follows for eqs. (45) that

$$\varphi_1 = \pi/2\,, \qquad \varphi_2 = \varphi\,, \qquad \varphi_3 = qD\,. \tag{59}$$

So the matrix of transformation (44) is simplified to

$$U_{\text{II}} = \exp[i\chi]\exp[-i\sigma_y qD/2]\exp[i\sigma_z\varphi/2]\,. \tag{60}$$

Here the index II indicates that we assume the helical field being inserted in the beam path II of the interferometer, whereas in beam path I we insert a homogeneous field with the same φ and a material with the same χ:

$$U_{\text{I}} = \exp[i\chi]\exp[i\sigma_z\varphi/2]\,. \tag{61}$$

Therefore, the spinor in the final state is

$$\begin{aligned}\psi_f &= \psi_0^{\text{I}} + \psi_0^{\text{II}} = \\ &= \tfrac{1}{2}\big(\exp[i\chi]\exp[i\sigma_z\varphi/2] + \exp[i\chi]\exp[-i\sigma_y qD/2]\exp[i\sigma_z\varphi/2]\big)\,\psi_i\,,\end{aligned} \tag{62}$$

where ψ_i denotes the wave function of the forward beam in the initial state without phase shifters. Rewriting (62) in the form

$$\psi_f = \tfrac{1}{2}\big(1+\exp[-i\sigma_y qD/2]\big)\exp[i\chi]\exp[i\sigma_z\varphi/2]\,\psi_i \tag{63}$$

and calculating the intensity

$$\begin{aligned}I_f &= \psi_f^\dagger\,\psi_f = \\ &= \tfrac{1}{4}\big[\psi_i^\dagger\exp[-i\sigma_z\varphi/2]\big(1+\exp[i\sigma_y qD/2]\big)\big(1+\exp[-i\sigma_y qD/2]\big)\exp[i\sigma_z\varphi/2]\,\psi_i\big]\end{aligned} \tag{64}$$

we finally obtain

$$I_f = \tfrac{1}{2} I_i[1 + \cos(qD/2)] \qquad \text{with} \qquad I_i = \psi_i^\dagger \psi_i\,. \tag{65}$$

This result shows, in complete analogy to the case of a homogeneous field, a variation of the final intensity with half of the rotation angle qD. Therefore in this case too the original wave function is obtained for rotation angles being even multiples of 2π.

4. – Conclusion.

Measuring the intensity and the polarization of the forward-diffracted beam behind a neutron interferometer with magnetic fields or materials as phase shifters, besides the test of the 4π rotational symmetry of spinors, statements on the magnetic domain structure can be expected. In this paper we have dealt with the large-domain case, where it is assumed that the domains have the same thickness as the whole sample. A further extension of the theory has to include smaller domains similar to the well-known neutron depolarization theory ([15,16]).

* * *

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● RIASSUNTO (•)

Con l'uso di un interferometro a neutroni è possibile studiare le proprietà di trasformazione degli spinori. Si studia il comportamento dei neutroni in campi magnetici omogenei ed elicoidali. Si calcola l'intensità e la polarizzazione del fascio di neutroni anteriore dopo l'interferometro, che è composto da un'onda non cambiata e da un'onda che è passata attraverso un materiale magnetico. Si fa attenzione ai campi adiabatici elicoidali. In tutti i casi si tiene conto dell'influenza del potenziale neutrone-nucleo.

(•) *Traduzione a cura della Redazione.*

Интерференционные явления и вращение спина нейтронов в магнитных материалах.

Резюме (*). - С помощью нейтронного интерферометра оказывается возможным исследовать свойства преобразований спиноров. Рассматривается поведение нейтронов в однородных и спиральных магнитных полях. Вычисляются интенсивность и поляризация прямого нейтронного пучка после интерферометра, который складывает неизмененную волну и волну, которая прошла через маганитный материал. Особое внимание уделяется адиабатическим спиральным полям. Во всех случаях учитывается влияние нейтронно-ядерного потенциала.

(*) *Переведено редакцией.*

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